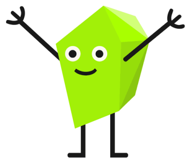
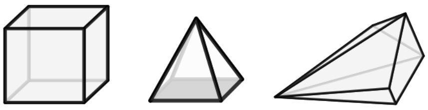
# Teacher's Handbook for "Polly's Journal"



There are an infinite number of convex polyhedra. Unlike the cube or the pyramid, which are both polyhedra, most of them do not have a name. Together with your students, you can change that. Adopt the polyhedra and give them names.

*Convex polyhedra* consist of:
- flat side surfaces
- straight edges
- outward pointing corners



The cube, the pyramid and an (still) unnamed polyhedron with six corners.

With the worksheets in "Polly's Journal", your students can get to know polyhedra while learning and practicing important content-related and process-related mathematical competences. In this handbook for teachers, you will find practical hints for the implementation of the project as well as useful links between the project and the curriculum as well as an overview of the learning prerequisites and objectives. In addition, a solution sheet is provided for the journal. At the end of this handbook, you will also find a feedback form for teachers. The email address for any feedback, as well as questions and comments is: schule@polytopia.eu.

## Contents of this Handbook

## Polly's Journal as a guide for the math class

In order to connect the project "Adopt a Polyhedron" to a standard math class for ages 10 to 14, we have developed the learning packet "Polly's Journal". Learning journals are a major component of dialogical learning. The journal serves as a guide for the students where the learning process is simultaneously presented and individually controlled.

The journal contains activities of various kinds, such as reading, writing, drawing, matching, measuring and arithmetic. Through these exercises, the students enter into an active dialogue with the material. The booklet is designed so that the learners can work through it relatively quickly and thus an immediate experience of learning success is made possible. While working through the packet, students have the possibility for self-reflection and to do two differentiation tasks.

The journal is divided into four sections. First, the concept of polyhedra is introduced and practiced in tasks for recognizing and drawing polyhedra and non-polyhedral bodies.  The next section deals with polyhedral nets. It starts with the already known cube net and is extended to general nets of polyhedra.

In the third section, the class builds a polyhedron model together. The students are divided into small groups and each group receives a different side surface of the polyhedron.  The edge lengths of the side surfaces are to be increased by a fixed factor, which is either provided by the teacher or determined by the students themselves. Here is a more advanced challenge for the students is to discover the quadratic relationship between edge lengths and the surface area. While the individual enlarged side surfaces are being assembled into a polyhedron, the students collect suggested names for the polyhedron. Upon completion of the model, a vote is taken to determine a name. Additionally, there is the possibility that every student adopts a separate polyhedron. You can download an entire class set of polyhedra that is generated from our database, resulting in a new set with each download.

Now that the students have dealt extensively with polyhedra and even adopted one, they are invited in the last section to take a research-based look at the subject. The usual course of mathematics education is turned upside down, because now it's about developing interesting and possibly unsolvable questions. The students take on the role of mathematical researchers to improve their overall understanding of polyhedra.

## Learning objectives

The main learning objectives of this packet are for students to understand the relationship between space and form, to understand different mathematical representations of a concept and to calculate area by measuring and arithmetic.

The convex polyhedra are explored in different geometric representations. The definition of convex polyhedra is presented by visual examples. Recognition and determination of convex polyhedra is first practiced on the basis of their three defining properties (flat side surfaces, straight corners, and outward pointing corners). Geometric bodies are analyzed according to these criteria and students must determine whether or not they are convex polyhedra.

Subsequently, polyhedra and polyhedral nets are associated with each other. In the process of assembling the two-dimensional polyhedral net into a three-dimensional model by hand, the representation is changed by the learners themselves. Through this action, the students also gain insight into how the same form can be perceived in different dimensions. The interactive visualization tool on the website offers another form of representation. For the students with access to VR glasses, another visualization experience is available to add even more perspective.

On the website, students find their personal polyhedra in an interactive, computer-aided, digital representation. Students can change the color of their polyhedron or change the perspective by switching the corner, edge or side surface viewers on and off. The students have the opportunity to give their personal polyhedron a name as well. The symbolic representation is therefore chosen by the students and is not given, as is usual in mathematics and the natural sciences. In this way, students learn that it is possible to participate in shaping science. This change in the perception of mathematics is one of the main goals of our project.

Finally, during the construction of the enlarged polyhedron, the lengths and angles of the side surfaces must be measured. Here, the handling of a protractor and its different scales is practiced. The measured quantities are entered into a table with reasonable accuracy. When enlarging the pieces, the lengths of the edges are multiplied but not the angles. Using a protractor will again be practiced when the students are drawing the enlarged side surface piece onto the cardboard.

## Learning prerequisites:

The students do not need to be completely comfortable with the prerequisites listed below. Most of these are repeated in "Polly's Journal", giving students a chance to practice and recall. The central prerequisites are:

- Measuring polygon edges
- Measuring and drawing angles
- Handling units of measure, such as centimeters and degrees
- Drawing of triangles and other polygons
- Multiplication of decimal numbers with an integer factor
- The net of the cube

# Practical information for preparation:

## Print

- Learning Journal "Polly's Journal", one copy per student
- A class polyhedron, three times
- Set of polyhedral nets (if possible printed on stronger paper)
- Solution sheet for hanging in the classroom

## Need

- Cardboard (if possible in different colors)
- Tape (if the class polyhedron is enlarged, the gluing tabs are cut off and needs to be assembled with tape)

## Plan

- Remind students to bring scissors, glue and a protractor.
- Do the students have active and accessible email addresses?
- Consider whether a factor for the enlargement of the side surfaces is given or determined by the students themselves. (Pay attention to the dimensions of the cardboard and what is feasible).
- Number the side surfaces of the class polyhedron on all three sheets. Cut one of the nets apart, also cutting off the grey gluing tabs.

# About the project, "Adopt a Polyhedron"

The project "Adopt a Polyhedron" is part of the public relations work of the Collaborative Research Center "Discretization in Dynamics and Geometry", which is funded by grants from the Deutschen Forschungsgemeinschaft (DFG) and is primarily involved with the structure and applications of discrete mathematics. Mathematicians from the Technische Universität Berlin, the Technische Universität München, and the Freie Universität Berlin are investigating the discretization of differential geometry and dynamic systems. Discrete in a mathematical context means distinguishable. For example, the four corners of a square are clearly separated while a circle could be understood as a polygon with an infinite number of indefinable corners. Three-dimensional polyhedra, with their well-defined corners, edges, and side surfaces belong to the classical research field of discrete geometry.

# Goals

The goal of this project is to build models of "all" polyhedra in a collective endeavor. To accomplish this, we have initially released all polyhedra with up to nine vertices for adoption. It is not possible to realize all polyhedra, as there are an infinite number of them, but everyone can help bring as many as possible to life by adopting their own individual polyhedron, giving it a name and then building a model,

In particular, we would like to invite students to actively participate in mathematics by focusing on the construction of geometric models. Modeling has long been a central discipline in (university) mathematics and has in recent decades been phased out by visualization with computers. However, the manual assembling of a model allows engagement and a deeper understanding of mathematics beyond abstract ideas.

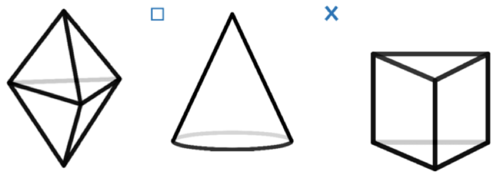
# Citizen Art

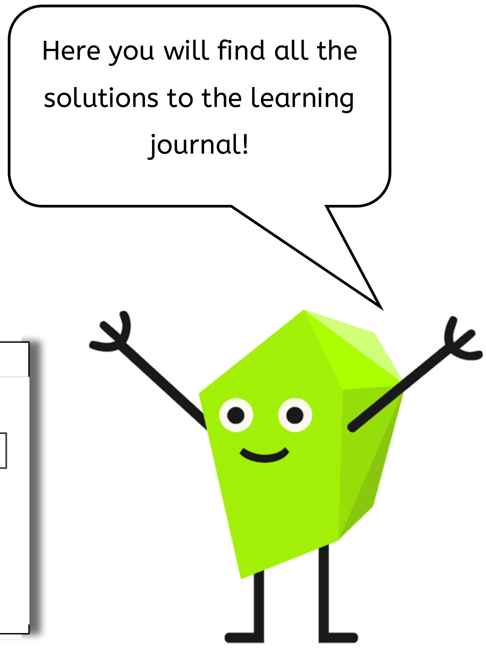
Lately, there has been an increasing effort in Citizen Science to actively engage the public in scientific research. As mathematicians, we also wish to offer interested people an opportunity to participate. Since the construction of models also has a creative and individual aspect and we want to emphasize the relationship between mathematics and art, we characterize our project under the term *Citizen Art*.

# Solutions

Mark which bodies are polyhedra:

× □ ×

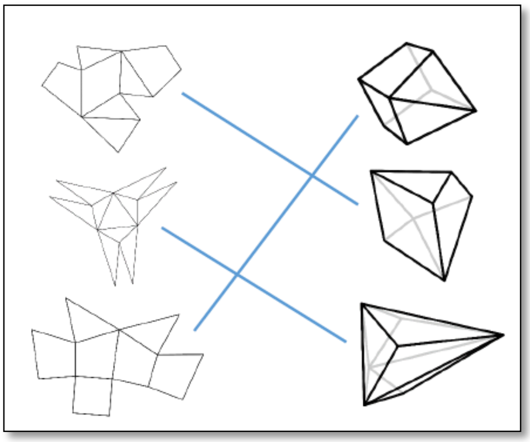





Fill in the blanks with the correct words from the box.

| straight, ball, round, inward, outward, prism, flat |
|---|

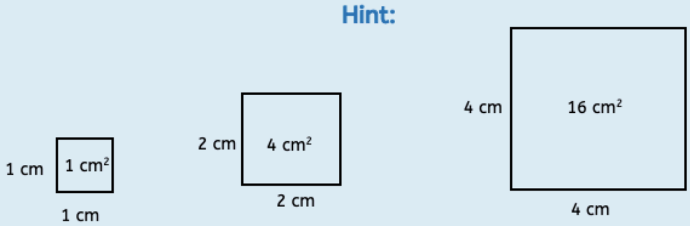
Polyhedra from Polly's family are bodies made from ____flat______ surfaces, ___straight____ edges and _____outward_______ corners. The cube and the pyramid both belong to Polly's family. The _____ball_____ is not a polyhedron.



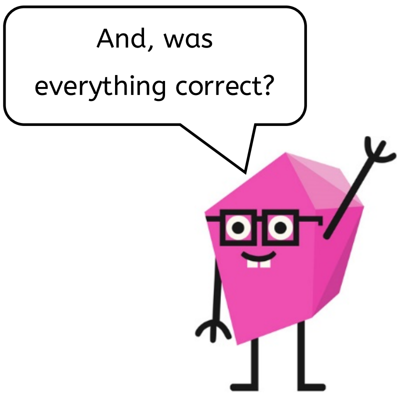
Is it a polyhedron? Mark the box if it is and explain why or why not.

| | | |
|---|---|---|
| ×  | ×  | □  |
| All faces are flat, all edges are straight, and all corners point outward. | All faces are flat, all edges are straight, and all corners point outward. | Two faces are not flat, and one edges is rounded. |

**Hint:**




If the edge length doubles,
then the surface area increases by a factor of __4__.



# Feedback:

We would like to know your thoughts on the project "Adopt a Polyhedron" and kindly ask you to complete this questionnaire. You can either copy these questions into an email and send them to schule@polytopia.eu, or by mail to: Anna Maria Hartkopf, Institut für Mathematik, Freie Universität Berlin, Arnimallee 2, 14195 Berlin.

1. In which grade / age level did you implement the project?

2. In which country / city is your school located?

3. How would you describe the performance of your students? Did the learning journal meet their learning level?

4. How much time did you plan for the project and how long did it actually take?

5. Did you complete the whole packet or only selected sections? If yes, which sections?

6. What worked well?

7. What did not work as well?

8. Which mathematical skills have your students newly learned or strengthened?

9. Were the students able to grasp a new view of mathematics through the project? If yes, how so?

10. Do you have any suggestions, tips or comments for improvement?